FYJC - MATHEMATICS & STATISTICS

HIGHLIGHTS

- ✓ *Solution to all questions*
- ✓ *solutions are put in way the student is expected to reproduce in the exam*
- ✓ *taught in the class room the same way as the solution are put up here . That makes the student to easily go through the solution & prepare him/herself when he/she sits back to revise and recall the topic at any given point of time .*
- ✓ *lastly, if student due to some unavoidable reasons , has missed the lecture , will not have to run here and there to update his/her notes .*
- ✓ *however class room lectures are must for easy passage of understanding & learning the minuest details of the given topic*

PAPER - I

DETERMINANTS

Compiled & Conducted @ JKSC

Q SET - 1
CRAMER'S RULE

01. $2x - y + 3z = 9$

$x + y + z = 6$

$x - y + z = 2$ SS : $\{1,2,3\}$

02. $x + y + z = 6$

$x - y + z = 2$

$2x + y - z = 1$ SS : $\{1,2,3\}$

03. $x - y + z = 4$

$2x + y - 3z = 0$

$x + y + z = 2$ SS : $\{2,-1,1\}$

04. $x + y + z = 2$

$x + 2y + z = 1$

$5x + y + z = 6$ SS : $\{1,-1,2\}$

05. $x + y + z = 45$

$z = x + 8$

$x + z = 2y$ SS : $\{11,15,19\}$

06. $\dfrac{1}{x} + \dfrac{1}{y} + \dfrac{1}{z} = 2$

$\dfrac{1}{x} - \dfrac{2}{y} + \dfrac{1}{z} = 3$

$\dfrac{2}{x} - \dfrac{1}{y} + \dfrac{3}{z} = -1$ SS : $\{1,-1,2\}$

07. $\sin x + \cos y + \tan z = 3$

$2\sin x + \cos y + \tan z = 4$

$3\sin x + 4\cos y - 2\tan z = 5$

SS : $\{\pi/2, 0, \pi/4\}$

08. $5e^x + 4\log_{10}y - 3\sqrt{z} = 1$

$4e^x + 3\log_{10}y - 2\sqrt{z} = 2$

$e^x + 2\log_{10}y - \sqrt{z} = 1$

SS $\{ 0 , 100 , 16 \}$

09. sum of 3 numbers is 2 . If twice the second number is added to sum of first and third we get 1 . On adding sum of second and third number to 5 times the first number we get 6 . Find the three numbers

SS : $\{1,-1,2\}$

PROPERTIES OF DETERMINANTS

Q SET - 2

WITHOUT EXPANSION SHOW THAT

01. $\begin{vmatrix} 1 & 1 & x \\ 1 & x & x^2 \\ 1 & x^2 & x^3 \end{vmatrix} = 0$

02. $\begin{vmatrix} x+a & x+b & x+c \\ y+a & y+b & y+c \\ z+a & z+b & z+c \end{vmatrix} = 0$

03. $\begin{vmatrix} x-y & x+y & x \\ z-x & z+x & z \\ y-z & y+z & y \end{vmatrix} = 0$

04. $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = 0$

05. $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix} = 0$

06. $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ y+z & z+x & x+y \end{vmatrix} = 0$

07. $\begin{vmatrix} 1 & xy & z(x+y) \\ 1 & yz & x(y+z) \\ 1 & zx & y(z+x) \end{vmatrix} = 0$

08. $\begin{vmatrix} xa & yb & zc \\ a^2 & b^2 & c^2 \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} x & y & z \\ a & b & c \\ bc & ca & ab \end{vmatrix}$

09. $\begin{vmatrix} lp & mq & nr \\ p^2 & q^2 & r^2 \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} l & m & n \\ p & q & r \\ qr & pr & pq \end{vmatrix}$

10. $\begin{vmatrix} bc & a & a^2 \\ ca & b & b^2 \\ ab & c & c^2 \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$

11. $\begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & c^2 & c^3 \end{vmatrix} = \begin{vmatrix} yz & x & x^2 \\ xz & y & y^2 \\ xy & z & z^2 \end{vmatrix}$

12. $\begin{vmatrix} b^2 + c^2 & ab & ac \\ ab & c^2 + a^2 & bc \\ ac & bc & a^2 + b^2 \end{vmatrix}$

$= \begin{vmatrix} b^2 + c^2 & b^2 & c^2 \\ a^2 & c^2 + a^2 & c^2 \\ a^2 & b^2 & a^2+b^2 \end{vmatrix}$

13. $\begin{vmatrix} b^2c^2 & bc & b + c \\ c^2a^2 & ca & c + a \\ a^2b^2 & ab & a + b \end{vmatrix} = 0$

14. $\begin{vmatrix} x + y & y + z & z + x \\ z + x & x + y & y + z \\ y + z & z + x & x + y \end{vmatrix} = 2\begin{vmatrix} x & y & z \\ z & x & y \\ y & z & x \end{vmatrix}$

15. $\begin{vmatrix} a + b & b + c & c + a \\ x + y & y + z & z + x \\ p + q & q + r & r + p \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix}$

16. $\begin{vmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{vmatrix} = 0$

17. $\begin{vmatrix} 0 & x - y & y - z \\ y - x & 0 & z - x \\ z - y & x - z & 0 \end{vmatrix} = 0$

18. $\begin{vmatrix} 11 & 4 & 10 \\ 2 & 7 & 6 \\ 5 & 1 & 4 \end{vmatrix} = 3\begin{vmatrix} 5 & 2 & 3 \\ 11 & 5 & 2 \\ 10 & 4 & 6 \end{vmatrix}$

PROPERTIES OF DETERMINANTS

Q SET - 3

WITHOUT EXPANDING AS FAR AS POSSIBLE

01. $\begin{vmatrix} a^2+ 2a & 2a + 1 & 1 \\ 2a + 1 & a + 2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a - 1)^3$

02. $\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = (x - y)(y - z)(z - x)$

03. $\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = (a - b)(b - c)(c - a)$

04. $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = abc(a-b)(b-c)(c-a)$

05. $\begin{vmatrix} x & p & q \\ p & x & q \\ p & q & x \end{vmatrix} = (x - p)(x - q)(x + p + q)$

06. $\begin{vmatrix} a & b+c & a^2 \\ b & c+a & b^2 \\ c & a+b & c^2 \end{vmatrix} = -(a-b)(b-c)(c-a)(a+b+c)$

07. $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$

08. $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$

09. $\begin{vmatrix} 1 & x & x^2 - yz \\ 1 & y & y^2 - zx \\ 1 & z & z^2 - xy \end{vmatrix} = 0$

10. $\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$

$= (a-b)(b-c)(c-a)(ab+bc+ca)$

11. $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^3 & y^3 & z^3 \end{vmatrix} = (x-y)(y-z)(z-x)(x+y+z)$

12. $\begin{vmatrix} y+z & z & y \\ z & z+x & x \\ y & x & x+y \end{vmatrix} = 4xyz$

13. **WITHOUT EXPANSION , PROVE**

$$\begin{vmatrix} 6 & 1 & 2 \\ 7 & 2 & 3 \\ 2 & 3 & -4 \end{vmatrix} + \begin{vmatrix} 2 & 4 & 7 \\ 3 & 4 & 2 \\ -4 & 3 & 3 \end{vmatrix} = 5\begin{vmatrix} 2 & 2 & 7 \\ 3 & 1 & 2 \\ -4 & 1 & 3 \end{vmatrix}$$

14. **WITHOUT EXPANSION , PROVE**

$$\begin{vmatrix} 2 & -3 & 4 \\ 5 & -6 & 2 \\ -3 & 1 & 3 \end{vmatrix} + \begin{vmatrix} 2 & 5 & -3 \\ 5 & 8 & 5 \\ 4 & 2 & 3 \end{vmatrix} = -2\begin{vmatrix} 2 & 4 & 1 \\ 5 & 2 & 1 \\ -3 & 3 & 3 \end{vmatrix}$$

Q SET - 4

CONSISTENCY OF EQUATIONS

01. $x + y = 3$; $5x + 6y = 17$; $2x - 3y = k$ are consistent . Find k

02. $3x + y = 2$; $kx + 2y = 3$; $2x - y = -3$ are consistent . Find k

03. $x + 3y + 2 = 0$; $4y + 2x = k$; $x - 2y = 3k$. are consistent . Find k

04. $2x - y + 3 = 0$; $7x - 2y + 2 = 0$; $kx - y - 1 = 0$ are consistent . Find k & SS

05. $3x + y = 2$; $kx + 2y = 3$; $2x - y = -3$. Find k if the lines are concurrent . Hence find the point of concurrency

06. $kx + 3y + 4 = 0$; $x + ky + 3 = 0$; $3x + 4y + 5 = 0$ are consistent . Find k . Hence find common solution set for smallest value of k

07. $(k-2)x + (k-1)y = 17$
$(k-1)x + (k-2)y = 18$
$x + y = 5$ are consistent . Find k

08. $(k+1)x + (k-1)y + (k-1) = 0$;
$(k-1)x + (k+1)y + (k-1) = 0$;
$(k-1)x + (k-1)y + (k+1) = 0$
are consistent . Find k

09. $x + y + k = 0$; $kx + 2ky + 6 = 0$; $x + ky + 1 = 0$ are consistent . Find k

10. $(k+2)x + 2y = 6$; $5x + (k-1)y = 6$; $5x + 2y = k + 3$. are consistent . Find k

11. $2kx - 3y = 8$; $4x - (k + 1)y = 8$; $4x - 3y = 3k + 2$. are consistent . Find k

12. if the equations $ax + by + c = 0$; $cx + ay + b = 0$; and $bx + cy + a = 0$ are consistent in x and y then show that $a^3 + b^3 + c^3 = 3abc$

13. if $l = \frac{y - 1}{x}$; $m = \frac{1 - x}{y}$; $n = x - y$ are consistent in x and y then prove : $l + m + n + lmn = 0$

14. Show that the equations

$x + ay + (a^2 - bc) = 0$;

$x + by + (b^2 - ca) = 0$

$x + cy + (c^2 - ab) = 0$

are consistent for all a , b , c

QSET 5

AREA OF TRIANGLE

Q1. Find area of triangle with vertices as

01. (4,5) ; (0,7) ; (−1,1)

02. (3,6) ; (−1,3) ; (2,−1)

03. (1,1) , (3,7) , (10,8)

Q2.

01. Find k if the area of the triangle whose vertices are A (4 , k) ; B (−5 , −7); C(−4 , 1) is 38 sq. units

02. Find k if the area of the triangle whose vertices are P (k , −4) ; Q (1 ,−2) ; R (4 ,− 5) is $^{15}/_{2}$ sq. units

03. Find k if the area of the triangle whose vertices are P (3 , −5) ; Q (−2 , k) ; R (1 , 4) is 33/2 sq. units

Q3. Find area of Quadrilateral whose vertices are A (2 , 1) ; B (2 , 3) ; C(−2 , 2) ; D(−1 , 0)

Q4.

Using determinants show that the following set of points are collinear

01. A(3,1) ; B(4,2) ; C(5, 3)

02. A(1,−2) ; B(3,1) ; C(5,4)

03. A(3,7) ; B(4,−3) ; C(5,−13)

04.

points A(a,0) ; B(0,b) , C(1,1) are collinear .

Prove : $\frac{1}{a} + \frac{1}{b} = 1$

05

if points A(a ,b) ;B(c , d) ; C(a − c , b − d) are collinear then prove that $ad - bc = 0$

SOLUTION - QSET 1

CRAMER'S RULE

01.

$$\begin{aligned} 2x - y + 3z &= 9 \\ x + y + z &= 6 \\ x - y + z &= 2 \end{aligned}$$

$$D = \begin{array}{c} + \quad - \quad + \\ \begin{vmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix} \end{array} \begin{aligned} &= 2(1+1)+1(1-1)+3(-1-1) \\ &= 2(2) + 1(0) + 3(-2) \\ &= 4-6 \\ &= -2 \end{aligned}$$

$$Dx = \begin{array}{c} + \quad - \quad + \\ \begin{vmatrix} 9 & -1 & 3 \\ 6 & 1 & 1 \\ 2 & -1 & 1 \end{vmatrix} \end{array} \begin{aligned} &= 9(1+1)+1(6-2)+3(-6-2) \\ &= 9(2) + 1(4) + 3(-8) \\ &= 18 + 4 - 24 \\ &= 22 - 24 \\ &= -2 \end{aligned}$$

$$Dy = \begin{array}{c} + \quad - \quad + \\ \begin{vmatrix} 2 & 9 & 3 \\ 1 & 6 & 1 \\ 1 & 2 & 1 \end{vmatrix} \end{array} \begin{aligned} &= 2(6-2)-9(1-1)+3(2-6) \\ &= 2(4) - 9(0) + 3(-4) \\ &= 8-12 \\ &= -4 \end{aligned}$$

$$Dz = \begin{array}{c} + \quad - \quad + \\ \begin{vmatrix} 2 & -1 & 9 \\ 1 & 1 & 6 \\ 1 & -1 & 2 \end{vmatrix} \end{array} \begin{aligned} &= 2(2+6)+1(2-6)+9(-1-1) \\ &= 2(8) + 1(-4) + 9(-2) \\ &= 16 - 4 - 18 \\ &= -6 \end{aligned}$$

$$x = \frac{Dx}{D} = \frac{-2}{-2} = 1 \quad ; \quad y = \frac{Dy}{D} = \frac{-4}{-2} = 2 \quad ; \quad z = \frac{Dz}{D} = \frac{-6}{-2} = 3$$

SS : {1,2,3}

02.

$$\begin{aligned} x + y + z &= 6 \\ x - y + z &= 2 \\ 2x + y - z &= 1 \end{aligned}$$

$$D = \begin{array}{c} + \quad - \quad + \\ \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 1 & -1 \end{vmatrix} \end{array} \begin{aligned} &= 1(1-1)-1(-1-2)+1(1+2) \\ &= 1(0) - 1(-3) + 1(3) \\ &= 3 + 3 \\ &= 6 \end{aligned}$$

$$Dx = \begin{array}{c} + \quad - \quad + \\ \begin{vmatrix} 6 & 1 & 1 \\ 2 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix} \end{array} \begin{aligned} &= 6(1-1)-1(-2-1)+1(2+1) \\ &= 6(0) - 1(-3) + 1(3) \\ &= 3 + 3 \\ &= 6 \end{aligned}$$

$$Dy = \begin{array}{c} + \quad - \quad + \\ \begin{vmatrix} 1 & 6 & 1 \\ 1 & 2 & 1 \\ 2 & 1 & -1 \end{vmatrix} \end{array} \begin{aligned} &= 1(-2-1)-6(-1-2)+1(1-4) \\ &= 1(-3) - 6(-3) + 1(-3) \\ &= -3 + 18 - 3 \\ &= 12 \end{aligned}$$

$$Dz = \begin{array}{c} + \quad - \quad + \\ \begin{vmatrix} 1 & 1 & 6 \\ 1 & -1 & 2 \\ 2 & 1 & 1 \end{vmatrix} \end{array} \begin{aligned} &= 1(-1-2)-1(1-4)+6(1+2) \\ &= 1(-3) - 1(-3) + 6(3) \\ &= -3 + 3 + 18 \\ &= 18 \end{aligned}$$

$$x = \frac{Dx}{D} = \frac{6}{6} = 1 \quad ; \quad y = \frac{Dy}{D} = \frac{12}{6} = 2 \quad ; \quad z = \frac{Dz}{D} = \frac{18}{6} = 3$$

SS : {1,2,3}

03.

$$\begin{aligned} x - y + z &= 4 \\ 2x + y - 3z &= 0 \\ x + y + z &= 2 \end{aligned}$$

$$D = \begin{array}{c} + \quad - \quad + \\ \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{vmatrix} \end{array} \begin{aligned} &= 1(1+3)+1(2+3)+1(2-1) \\ &= 1(4) + 1(5) + 1(1) \\ &= 4 + 5 + 1 \\ &= 10 \end{aligned}$$

$$Dx = \begin{array}{c} + \quad - \quad + \\ \begin{vmatrix} 4 & -1 & 1 \\ 0 & 1 & -3 \\ 2 & 1 & 1 \end{vmatrix} \end{array} \begin{aligned} &= 4(1+3)+1(0+6)+1(0-2) \\ &= 4(4) + 1(6) + 1(-2) \\ &= 16 + 6 - 2 \\ &= 20 \end{aligned}$$

$$Dy = \begin{array}{c} + \quad - \quad + \\ \begin{vmatrix} 1 & 4 & 1 \\ 2 & 0 & -3 \\ 1 & 2 & 1 \end{vmatrix} \end{array} \begin{aligned} &= 1(0+6)-4(2+3)+1(4-0) \\ &= 1(6) - 4(5) + 1(4) \\ &= 6 - 20 + 4 \\ &= -10 \end{aligned}$$

$$Dz = \begin{vmatrix} 1 & -1 & 4 \\ 2 & 1 & 0 \\ 1 & 1 & 2 \end{vmatrix} \begin{aligned} &= 1(2-0)+1(4-0)+4(2-1) \\ &= 1(2) + 1(4) + 4(1) \\ &= 2 + 4 + 4 \\ &= 10 \end{aligned}$$

$$x = \frac{Dx}{D} \quad ; \quad y = \frac{Dy}{D} \quad ; \quad z = \frac{Dz}{D}$$

$$= \frac{20}{10} \qquad = \frac{-10}{10} \qquad = \frac{10}{10}$$

$$= 2 \qquad = -1 \qquad = 1$$

SS : {2,-1,1}

04. $x + y + z = 2$

$x + 2y + z = 1$

$5x + y + z = 6$

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 5 & 1 & 1 \end{vmatrix} \begin{aligned} &= 1(2-1)-1(1-5)+1(1-10) \\ &= 1(1) - 1(-4) + 1(-9) \\ &= 1 + 4 - 9 \\ &= -4 \end{aligned}$$

$$Dx = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 6 & 1 & 1 \end{vmatrix} \begin{aligned} &= 2(2-1)-1(1-6)+1(1-12) \\ &= 2(1) - 1(-5) + 1(-11) \\ &= 2 + 5 - 11 \\ &= -4 \end{aligned}$$

$$Dy = \begin{vmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 5 & 6 & 1 \end{vmatrix} \begin{aligned} &= 1(1-6)-2(1-5)+1(6-5) \\ &= 1(-5) - 2(-4) + 1(1) \\ &= -5 + 8 + 1 \\ &= 4 \end{aligned}$$

$$Dz = \begin{vmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 5 & 1 & 6 \end{vmatrix} \begin{aligned} &= 1(12-1)-1(6-5)+2(1-10) \\ &= 1(11) - 1(1) + 2(-9) \\ &= 11 - 1 - 18 \\ &= -8 \end{aligned}$$

$$x = \frac{Dx}{D} \quad ; \quad y = \frac{Dy}{D} \quad ; \quad z = \frac{Dz}{D}$$

$$= \frac{-4}{-4} \qquad = \frac{4}{-4} \qquad = \frac{-8}{-4}$$

$$= 1 \qquad = -1 \qquad = 2$$

SS : {1,-1,2}

05. $x + y + z = 45 \qquad x + y + z = 45$

$z = x + 8 \qquad x - z = -8$

$x + z = 2y \qquad x - 2y + z = 0$

$$\begin{array}{cc} & + \quad - \quad + \\ D = & \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & -2 & 1 \end{vmatrix} \end{array} \begin{aligned} &= 1(0-2)-1(1+1)+1(-2-0) \\ &= 1(-2) - 1(2) + 1(-2) \\ &= -2 - 2 - 2 \\ &= -6 \end{aligned}$$

$$\begin{array}{cc} & + \quad - \quad + \\ Dx = & \begin{vmatrix} 45 & 1 & 1 \\ -8 & 0 & -1 \\ 0 & -2 & 1 \end{vmatrix} \end{array} \begin{aligned} &= 45(0-2)-1(-8+0)+1(16-0) \\ &= 45(-2) - 1(-8) + 1(16) \\ &= -90 + 8 + 16 \\ &= -66 \end{aligned}$$

$$\begin{array}{cc} & + \quad - \quad + \\ Dy = & \begin{vmatrix} 1 & 45 & 1 \\ 1 & -8 & -1 \\ 1 & 0 & 1 \end{vmatrix} \end{array} \begin{aligned} &= 1(-8+0)-45(1+1)+1(0+8) \\ &= 1(-8) - 45(2) + 1(8) \\ &= -8 - 90 + 8 \\ &= -90 \end{aligned}$$

$$\begin{array}{cc} & + \quad - \quad + \\ Dz = & \begin{vmatrix} 1 & 1 & 45 \\ 1 & 0 & -8 \\ 1 & -2 & 0 \end{vmatrix} \end{array} \begin{aligned} &= 1(0-16)-1(0+8)+45(-2-0) \\ &= 1(-16) - 1(8) + 45(-2) \\ &= -16 - 8 - 90 \\ &= -114 \end{aligned}$$

$$x = \frac{Dx}{D} \quad ; \quad y = \frac{Dy}{D} \quad ; \quad z = \frac{Dz}{D}$$

$$= \frac{-66}{-6} \qquad = \frac{-90}{-6} \qquad = \frac{-114}{-6}$$

$$= 11 \qquad = 15 \qquad = 19$$

SS : {11,15,19}

06. $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 2$

$\frac{1}{x} - \frac{2}{y} + \frac{1}{z} = 3$

$\frac{2}{x} - \frac{1}{y} + \frac{3}{z} = -1$

Put $\frac{1}{x} = a \; ; \; \frac{1}{y} = b \; ; \; \frac{1}{z} = c$

$$\begin{aligned} a + b + c &= 2 \\ a - 2b + c &= 3 \\ 2a - b + 3c &= -1 \end{aligned}$$

$$D = \begin{array}{c} \begin{matrix} + & - & + \end{matrix} \\ \begin{vmatrix} 1 & 1 & 1 \\ 1 & -2 & 1 \\ 2 & -1 & 3 \end{vmatrix} \end{array} \begin{aligned} &= 1(-6+1)-1(3-2)+1(-1+4) \\ &= 1(-5) - 1(1) + 1(3) \\ &= -5 - 1 + 3 \\ &= -3 \end{aligned}$$

$$D_a = \begin{array}{c} \begin{matrix} + & - & + \end{matrix} \\ \begin{vmatrix} 2 & 1 & 1 \\ 3 & -2 & 1 \\ -1 & -1 & 3 \end{vmatrix} \end{array} \begin{aligned} &= 2(-6+1)-1(9+1)+1(-3-2) \\ &= 2(-5) - 1(10) + 1(-5) \\ &= -10 - 10 - 5 \\ &= -25 \end{aligned}$$

$$D_b = \begin{array}{c} \begin{matrix} + & - & + \end{matrix} \\ \begin{vmatrix} 1 & 2 & 1 \\ 1 & 3 & 1 \\ 2 & -1 & 3 \end{vmatrix} \end{array} \begin{aligned} &= 1(9+1)-2(3-2)+1(-1-6) \\ &= 1(10) - 2(1) + 1(-7) \\ &= 10 - 2 - 7 \\ &= 1 \end{aligned}$$

$$D_c = \begin{array}{c} \begin{matrix} + & - & + \end{matrix} \\ \begin{vmatrix} 1 & 1 & 2 \\ 1 & -2 & 3 \\ 2 & -1 & -1 \end{vmatrix} \end{array} \begin{aligned} &= 1(2+3)-1(-1-6)+2(-1+4) \\ &= 1(5) - 1(-7) + 2(3) \\ &= 5 + 7 + 6 \\ &= 18 \end{aligned}$$

$$a = \frac{D_a}{D} = \frac{-25}{-3} = \frac{25}{3} \quad ; \quad b = \frac{D_b}{D} = \frac{1}{-3} = \frac{-1}{3} \quad ; \quad c = \frac{D_c}{D} = \frac{18}{-3} = -6$$

RESUBS.

$$\frac{1}{x} = \frac{25}{3} \quad ; \quad \frac{1}{y} = \frac{-1}{3} \quad ; \quad \frac{1}{z} = -6$$

$$x = \frac{3}{25} \quad ; \quad y = -3 \quad ; \quad z = \frac{-1}{6}$$

SS : $\{1,-1,2\}$

07. $$\begin{aligned} \sin x + \cos y + \tan z &= 3 \\ 2\sin x + \cos y + \tan z &= 4 \\ 3\sin x + 4\cos y - 2\tan z &= 5 \end{aligned}$$

let $\sin x = a$, $\cos y = b$, $\tan z = c$

$$\begin{aligned} a + b + c &= 3 \\ 2a + b + c &= 4 \\ 3a + 4b - 2c &= 5 \end{aligned}$$

$$D = \begin{array}{c} \begin{matrix} + & - & + \end{matrix} \\ \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 3 & 4 & -2 \end{vmatrix} \end{array} \begin{aligned} &= 1(-2-4)-1(-4-3)+1(8-3) \\ &= 1(-6) - 1(-7) + 1(5) \\ &= -6 + 7 + 5 \\ &= 6 \end{aligned}$$

$$D_a = \begin{array}{c} \begin{matrix} + & - & + \end{matrix} \\ \begin{vmatrix} 3 & 1 & 1 \\ 4 & 1 & 1 \\ 5 & 4 & -2 \end{vmatrix} \end{array} \begin{aligned} &= 3(-2-4)-1(-8-5)+1(16-5) \\ &= 3(-6) - 1(-13) + 1(11) \\ &= -18 + 13 + 11 \\ &= 6 \end{aligned}$$

$$D_b = \begin{array}{c} \begin{matrix} + & - & + \end{matrix} \\ \begin{vmatrix} 1 & 3 & 1 \\ 2 & 4 & 1 \\ 3 & 5 & -2 \end{vmatrix} \end{array} \begin{aligned} &= 1(-8-5)-3(-4-3)+1(10-12) \\ &= 1(-13) - 3(-7) + 1(-2) \\ &= -13 + 21 - 2 \\ &= 6 \end{aligned}$$

$$D_c = \begin{array}{c} \begin{matrix} + & - & + \end{matrix} \\ \begin{vmatrix} 1 & 1 & 3 \\ 2 & 1 & 4 \\ 3 & 4 & 5 \end{vmatrix} \end{array} \begin{aligned} &= 1(5-16)-1(10-12)+3(8-3) \\ &= 1(-11) - 1(-2) + 3(5) \\ &= -11 + 2 + 15 \\ &= 6 \end{aligned}$$

$$a = \frac{D_a}{D} = \frac{6}{6} = 1 \quad ; \quad b = \frac{D_b}{D} = \frac{6}{6} = 1 \quad ; \quad c = \frac{D_c}{D} = \frac{6}{6} = 1$$

RESUBS

$\sin x = 1$; $\cos y = 1$; $\tan z = 1$

$x = {}^{\pi}/_{2}$ $\quad y = 0$ $\quad z = {}^{\pi}/_{4}$

SS : $\{{}^{\pi}/_{2}, 0, {}^{\pi}/_{4}\}$

08. $5e^x + 4\log_{10}y - 3\sqrt{z} = 1$
$4e^x + 3\log_{10}y - 2\sqrt{z} = 2$
$e^x + 2\log_{10}y - \sqrt{z} = 1$

let $e^x = a$; $\log_{10}y = b$; $\sqrt{z} = c$

$5a + 4b - 3c = 1$
$4a + 3b - 2c = 2$
$a + 2b - c = 1$

$$D = \begin{array}{ccc} + & - & + \end{array}$$
$$D = \begin{vmatrix} 5 & 4 & -3 \\ 4 & 3 & -2 \\ 1 & 2 & -1 \end{vmatrix} \begin{aligned} &= 5(-3+4)-4(-4+2)-3(8-3) \\ &= 5(1) - 4(-2) - 3(5) \\ &= 5 + 8 - 15 \\ &= -2 \end{aligned}$$

$$D_a = \begin{vmatrix} 1 & 4 & -3 \\ 2 & 3 & -2 \\ 1 & 2 & -1 \end{vmatrix} \begin{aligned} &= 1(-3+4)-4(-2+2)-3(4-3) \\ &= 1(1) - 4(0) - 3(1) \\ &= 1 - 0 - 3 \\ &= -2 \end{aligned}$$

$$D_b = \begin{vmatrix} 5 & 1 & -3 \\ 4 & 2 & -2 \\ 1 & 1 & -1 \end{vmatrix} \begin{aligned} &= 5(-2+2)-1(-4+2)-3(4-2) \\ &= 5(0) - 1(-2) - 3(2) \\ &= 0 + 2 - 6 \\ &= -4 \end{aligned}$$

$$D_c = \begin{vmatrix} 5 & 4 & 1 \\ 4 & 3 & 2 \\ 1 & 2 & 1 \end{vmatrix} \begin{aligned} &= 5(3-4)-4(4-2)+1(8-3) \\ &= 5(-1) - 4(2) + 1(5) \\ &= -5 - 8 + 5 \\ &= -8 \end{aligned}$$

$a = \dfrac{D_a}{D}$; $b = \dfrac{D_b}{D}$; $c = \dfrac{D_c}{D}$

$= \dfrac{-2}{-2}$ $\quad = \dfrac{-4}{-2}$ $\quad = \dfrac{-8}{-2}$

$= 1$ $\quad = 2$ $\quad = 4$

RESUBS

$e^x = 1$; $\log_{10}y = 2$; $\sqrt{z} = 4$
$x = 0$; $y = 10^2$; $z = 16$
$y = 100$

SS { 0 , 100 , 16 }

09. sum of 3 numbers is 2 . If twice the second number is added to sum of first and third we get 1 . On adding sum of second and third number to 5 times the first number we get 6 . Find the three numbers

$x + y + z = 2$
$x + 2y + z = 1$
$5x + y + z = 6$ REFER SOLN OF (04)

WITHOUT EXPANSION SHOW THAT

SOLUTION - QSET 2

01. $\begin{vmatrix} 1 & 1 & x \\ 1 & x & x^2 \\ 1 & x^2 & x^3 \end{vmatrix} = 0$

LHS

Taking 'x' common from C_1

$= x \begin{vmatrix} 1 & 1 & 1 \\ 1 & x & x \\ 1 & x^2 & x^2 \end{vmatrix}$

$= x(0)$ ….. C_1 & C_2 are identical

$= 0$

02. $\begin{vmatrix} x+a & x+b & x+c \\ y+a & y+b & y+c \\ z+a & z+b & z+c \end{vmatrix} = 0$

LHS

$C_1 - C_2$, $C_2 - C_3$

$= \begin{vmatrix} a-b & b-c & x+c \\ a-b & b-c & y+c \\ a-b & b-c & z+c \end{vmatrix}$

Taking 'a – b' common from C_1 &
'b – c' common from C_2

$= (a-b)(b-c) \begin{vmatrix} 1 & 1 & x+c \\ 1 & 1 & y+c \\ 1 & 1 & z+c \end{vmatrix}$

$= (a-b)(b-c)\ (0)$

……. C_1 & C_2 are identical

$= 0$

03. $\begin{vmatrix} x-y & x+y & x \\ z-x & z+x & z \\ y-z & y+z & y \end{vmatrix} = 0$

LHS

$C_1 + C_2$

$= \begin{vmatrix} x-y & x+y & x \\ z-x & z+x & z \\ y-z & y+z & y \end{vmatrix}$

$= \begin{vmatrix} 2x & x+y & x \\ 2z & z+x & z \\ 2y & y+z & y \end{vmatrix}$

Taking '2' common from C_1

$= 2\begin{vmatrix} x & x+y & x \\ z & z+x & z \\ y & y+z & y \end{vmatrix}$

$= 2(0)$ ……. C_1 & C_3 are identical

$= 0$

04. $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = 0$

LHS

$C_1 + C_2$

$= \begin{vmatrix} a-c & b-c & c-a \\ b-a & c-a & a-b \\ c-b & a-b & b-c \end{vmatrix}$

Taking (–) common from C_1

$= -\begin{vmatrix} c-a & b-c & c-a \\ a-b & c-a & a-b \\ b-c & a-b & b-c \end{vmatrix}$

$=$ 0 …… C_1 & C_3 are identical

05. $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix} = 0$

LHS

$C_3 + C_2$

$$= \begin{vmatrix} 1 & a & a+b+c \\ 1 & b & a+b+c \\ 1 & c & a+b+c \end{vmatrix}$$

taking (a + b + c) common from C_3

$$= \begin{vmatrix} 1 & a & a+b+c \\ 1 & b & a+b+c \\ 1 & c & a+b+c \end{vmatrix}$$

$$= (a+b+c) \begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix}$$

$= (a + b + c)\ (0)$ ..... C_1 & C_3 are identical

06. $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ y+z & z+x & x+y \end{vmatrix} = 0$

LHS

$R_3 + R_2$

$$= \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x+y+z & x+y+z & x+y+z \end{vmatrix}$$

taking (x + y + z) common from R_3

$$= (x+y+z) \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ 1 & 1 & 1 \end{vmatrix}$$

$= (x + y + z)\ (0)$ ..... R_1 & R_3 are identical

07. $\begin{vmatrix} 1 & xy & z(x+y) \\ 1 & yz & x(y+z) \\ 1 & zx & y(z+x) \end{vmatrix} = 0$

LHS

$$= \begin{vmatrix} 1 & xy & xz+yz \\ 1 & yz & xy+xz \\ 1 & zx & yz+xy \end{vmatrix}$$

$C_3 + C_2$

$$= \begin{vmatrix} 1 & xy & xy+yz+zx \\ 1 & yz & xy+yz+zx \\ 1 & zx & xy+yz+zx \end{vmatrix}$$

taking (xy + yz + zx) common from C_3

$$= (xy+yz+zx) \begin{vmatrix} 1 & xy & 1 \\ 1 & yz & 1 \\ 1 & zx & 1 \end{vmatrix}$$

$= (xy + yz + zx)(0)$ ... C_1 & C_3 are identical

$= 0$

08. $\begin{vmatrix} xa & yb & zc \\ a^2 & b^2 & c^2 \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} x & y & z \\ a & b & c \\ bc & ca & ab \end{vmatrix}$

RHS

$C_1(a)$, $C_2(b)$, $C_3(c)$

$$= \frac{1}{abc} \begin{vmatrix} xa & yb & zc \\ a^2 & b^2 & c^2 \\ abc & abc & abc \end{vmatrix}$$

taking (abc) common from R_3

$$= \cancel{\frac{abc}{abc}} \begin{vmatrix} xa & yb & zc \\ a^2 & b^2 & c^2 \\ 1 & 1 & 1 \end{vmatrix} = \text{RHS}$$

09. $\begin{vmatrix} lp & mq & nr \\ p^2 & q^2 & r^2 \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} l & m & n \\ p & q & r \\ qr & pr & pq \end{vmatrix}$

RHS

$C_1(p)$, $C_2(q)$, $C_3(r)$

$$= \frac{1}{pqr} \begin{vmatrix} lp & mq & nr \\ p^2 & q^2 & r^2 \\ pqr & pqr & pqr \end{vmatrix}$$

taking (pqr) common from R_3

$$= \frac{\cancel{pqr}}{\cancel{pqr}} \begin{vmatrix} lp & mq & nr \\ p^2 & q^2 & r^2 \\ 1 & 1 & 1 \end{vmatrix} = \text{RHS}$$

10. $\begin{vmatrix} bc & a & a^2 \\ ca & b & b^2 \\ ab & c & c^2 \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$

LHS

$R_1(a)$, $R_2(b)$, $R_3(c)$

$$= \frac{1}{abc} \begin{vmatrix} abc & a^2 & a^3 \\ abc & b^2 & b^3 \\ abc & c^2 & c^3 \end{vmatrix}$$

taking (abc) common from C_1

$$= \frac{\cancel{abc}}{\cancel{abc}} \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} = \text{RHS}$$

11. $\begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & c^2 & c^3 \end{vmatrix} = \begin{vmatrix} yz & x & x^2 \\ xz & y & y^2 \\ xy & z & z^2 \end{vmatrix}$

RHS

$R_1(x)$, $R_2(y)$, $R_3(z)$

$$= \frac{1}{xyz} \begin{vmatrix} xyz & x^2 & x^3 \\ xyz & y^2 & y^3 \\ xyz & z^2 & z^3 \end{vmatrix}$$

taking (xyz) common from C_1

$$= \frac{\cancel{xyz}}{\cancel{xyz}} \begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix} = \text{RHS}$$

12. $\begin{vmatrix} b^2 + c^2 & ab & ac \\ ab & c^2 + a^2 & bc \\ ac & bc & a^2 + b^2 \end{vmatrix}$

$$= \begin{vmatrix} b^2 + c^2 & b^2 & c^2 \\ a^2 & c^2 + a^2 & c^2 \\ a^2 & b^2 & a^2+b^2 \end{vmatrix}$$

LHS

$C_1(a)$, $C_2(b)$, $C_3(c)$

$$= \frac{1}{abc} \begin{vmatrix} a(b^2 + c^2) & ab^2 & ac^2 \\ a^2b & b(c^2 + a^2) & bc^2 \\ a^2c & b^2c & c(a^2 + b^2) \end{vmatrix}$$

taking a , b & c common from R_1 , R_2 & R_3 respectively

$$= \frac{\cancel{abc}}{\cancel{abc}} \begin{vmatrix} b^2 + c^2 & b^2 & c^2 \\ a^2 & c^2 + a^2 & c^2 \\ a^2 & b^2 & a^2+b^2 \end{vmatrix} = \text{RHS}$$

13. $\begin{vmatrix} b^2c^2 & bc & b + c \\ c^2a^2 & ca & c + a \\ a^2b^2 & ab & a + b \end{vmatrix} = 0$

$R_1(a)$, $R_2(b)$, $R_3(c)$

$$= \frac{1}{abc} \begin{vmatrix} ab^2c^2 & abc & ab + ac \\ bc^2a^2 & abc & bc + ab \\ a^2b^2c & abc & ac + bc \end{vmatrix}$$

taking (abc) common from C_1 & C_2

$$= \frac{\cancel{abc}}{\cancel{abc}} \begin{vmatrix} bc & 1 & ab + ac \\ ac & 1 & bc + ab \\ ab & 1 & ac + bc \end{vmatrix}$$

$C_3 + C_1$

$$= \begin{vmatrix} bc & 1 & ab + bc + ac \\ ac & 1 & ab + bc + ac \\ ab & 1 & ab + bc + ac \end{vmatrix}$$

taking (ab + bc + ac) common from C_3

$$= (ab + bc + ac) \begin{vmatrix} bc & 1 & 1 \\ ac & 1 & 1 \\ ab & 1 & 1 \end{vmatrix}$$

$= (ab + bc + ac)(0)$ ... C_2 & C_3 identical

$= 0 = \text{RHS}$

14. $\begin{vmatrix} x+y & y+z & z+x \\ z+x & x+y & y+z \\ y+z & z+x & x+y \end{vmatrix} = 2\begin{vmatrix} x & y & z \\ z & x & y \\ y & z & x \end{vmatrix}$

LHS

$C_1 - C_2$

$$= \begin{vmatrix} x-z & y+z & z+x \\ z-y & x+y & y+z \\ y-x & z+x & x+y \end{vmatrix}$$

$C_1 + C_3$

$$= \begin{vmatrix} 2x & y+z & z+x \\ 2z & x+y & y+z \\ 2y & z+x & x+y \end{vmatrix}$$

Taking '2' common from C_1

$$= 2\begin{vmatrix} x & y+z & z+x \\ z & x+y & y+z \\ y & z+x & x+y \end{vmatrix}$$

$C_3 - C_1$

$$= 2\begin{vmatrix} x & y+z & z \\ z & x+y & y \\ y & z+x & x \end{vmatrix}$$

$C_2 - C_3$

$$= 2\begin{vmatrix} x & y & z \\ z & x & y \\ y & z & x \end{vmatrix} = \text{RHS}$$

15. $\begin{vmatrix} a+b & b+c & c+a \\ x+y & y+z & z+x \\ p+q & q+r & r+p \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix}$

LHS

$C_1 - C_2$

$$= \begin{vmatrix} a-c & b+c & c+a \\ x-z & y+z & z+x \\ p-r & q+r & r+p \end{vmatrix}$$

$C_1 + C_3$

$$= \begin{vmatrix} 2a & b+c & c+a \\ 2x & y+z & z+x \\ 2p & q+r & r+p \end{vmatrix}$$

Taking '2' common from C_1

$$= 2\begin{vmatrix} a & b+c & c+a \\ x & y+z & z+x \\ p & q+r & r+p \end{vmatrix}$$

$C_3 - C_1$

$$= 2\begin{vmatrix} a & b+c & c \\ x & y+z & z \\ p & q+r & r \end{vmatrix}$$

$C_2 - C_3$

$$= 2\begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix} = \text{RHS}$$

16. $\begin{vmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{vmatrix} = 0$

$$\text{let } D = \begin{vmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{vmatrix}$$

$R \leftrightarrow C$

$$D = \begin{vmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{vmatrix}$$

Taking (–) common from C_1, C_2 & C_3

$$D = (-1)^3 \begin{vmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{vmatrix}$$

$D = -D$

$2D = 0 \quad \therefore D = 0$

17. $$\begin{vmatrix} 0 & x-y & y-z \\ y-x & 0 & z-x \\ z-y & x-z & 0 \end{vmatrix} = 0$$

let

$$D = \begin{vmatrix} 0 & x-y & y-z \\ y-x & 0 & z-x \\ z-y & x-z & 0 \end{vmatrix}$$

$R \leftrightarrow C$

$$D = \begin{vmatrix} 0 & y-x & z-y \\ x-y & 0 & x-z \\ y-z & z-x & 0 \end{vmatrix}$$

Taking (–) common from C_1 , C_2 & C_3

$$D = (-)^3 \begin{vmatrix} 0 & x-y & y-z \\ y-x & 0 & z-x \\ z-y & x-z & 0 \end{vmatrix}$$

$$D = -D$$

$$2D = 0 \qquad \therefore D = 0$$

18. $$\begin{vmatrix} 11 & 4 & 10 \\ 2 & 7 & 6 \\ 5 & 1 & 4 \end{vmatrix} = 3\begin{vmatrix} 5 & 2 & 3 \\ 11 & 5 & 2 \\ 10 & 4 & 6 \end{vmatrix}$$

$$LHS = \begin{vmatrix} 11 & 4 & 10 \\ 2 & 7 & 6 \\ 5 & 1 & 4 \end{vmatrix}$$

$R \leftrightarrow C$

$$= \begin{vmatrix} 11 & 2 & 5 \\ 4 & 7 & 1 \\ 10 & 6 & 4 \end{vmatrix}$$

$C_2 \leftrightarrow C_3$

$$= -\begin{vmatrix} 11 & 5 & 2 \\ 4 & 1 & 7 \\ 10 & 4 & 6 \end{vmatrix}$$

$R_1 \leftrightarrow R_2$

$$= +\begin{vmatrix} 4 & 1 & 7 \\ 11 & 5 & 2 \\ 10 & 4 & 6 \end{vmatrix}$$

$R_1 + R_2$

$$= +\begin{vmatrix} 15 & 6 & 9 \\ 11 & 5 & 2 \\ 10 & 4 & 6 \end{vmatrix}$$

Taking '3' common from R_1

$$= 3\begin{vmatrix} 5 & 2 & 3 \\ 11 & 5 & 2 \\ 10 & 4 & 6 \end{vmatrix} = RHS$$

PROPERTIES OF DETERMINANTS

Q SET - 3

WITHOUT EXPANDING AS FAR AS POSSIBLE

01. $\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a-1)^3$

$R_1 - R_2$, $R_2 - R_3$

$$= \begin{vmatrix} a^2-1 & a-1 & 0 \\ 2a-2 & a-1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$

Taking (a − 1) common from R_1 & R_2

$$= (a-1)^2 \begin{vmatrix} a+1 & 1 & 0 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$

$R_1 - R_2$

$$= (a-1)^2 \begin{vmatrix} a-1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$

Taking (a − 1) common from R_1

$$= (a-1)^3 \begin{vmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$

Expanding the determinant

$$= (a-1)^3 \left[1(1-0)\right]$$

$$= (a-1)^3$$

02. $\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = (x-y)(y-z)(z-x)$

$R_1 - R_2$, $R_2 - R_3$

$$= \begin{vmatrix} 0 & x-y & x^2-y^2 \\ 0 & y-z & y^2-z^2 \\ 1 & z & z^2 \end{vmatrix}$$

Taking (x − y) common from R_1 & (y − z) common from R_2

$$= (x-y)(y-z) \begin{vmatrix} 0 & 1 & x+y \\ 0 & 1 & y+z \\ 1 & z & z^2 \end{vmatrix}$$

$R_1 - R_2$

$$= (x-y)(y-z) \begin{vmatrix} 0 & 0 & x-z \\ 0 & 1 & y+z \\ 1 & z & z^2 \end{vmatrix}$$

Taking (x − z) common from R_1

$$= (x-y)(y-z)(x-z) \begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & y+z \\ 1 & z & z^2 \end{vmatrix}$$

Expanding the determinant

$$= (x-y)(y-z)(x-z)\left[1(0-1)\right]$$

$$= (x-y)(y-z)(x-z)\ (-1)$$

$$= (x-y)(y-z)(z-x)$$

03. $\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = (a - b)(b - c)(c - a)$

$R_1 - R_2$, $R_2 - R_3$

$$= \begin{vmatrix} 0 & a - b & bc - ca \\ 0 & b - c & ca - ab \\ 1 & c & ab \end{vmatrix}$$

$$= \begin{vmatrix} 0 & a - b & c(b - a) \\ 0 & b - c & a(c - b) \\ 1 & c & ab \end{vmatrix}$$

Taking (a – b) common from R_1 & (b – c) common from R_2

$$= (a - b)(b - c)\begin{vmatrix} 0 & 1 & -c \\ 0 & 1 & -a \\ 1 & c & ab \end{vmatrix}$$

$R_1 - R_2$

$$= (a - b)(b - c)\begin{vmatrix} 0 & 0 & a - c \\ 0 & 1 & -a \\ 1 & c & ab \end{vmatrix}$$

Taking (a – c) common from R_1

$$= (a - b)(b - c)(a - c)\begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & -a \\ 1 & z & ab \end{vmatrix}$$

$$= (a - b)(b - c)(a - c)\ \Big(1(0 - 1)\Big)$$

$$= (a - b)(b - c)(a - c)(-1)$$

$$= (a - b)(b - c)(c - a)$$

04. $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = abc(a-b)(b-c)(c-a)$

taking 'a ' , 'b' & 'c' common from C_1 , C_2 & C_3 respectively

$$= abc\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$$

$C_1 - C_2$, $C_2 - C_3$

$$= abc\begin{vmatrix} 0 & 0 & 1 \\ a - b & b - c & c \\ a^2 - b^2 & b^2 - c^2 & c^2. \end{vmatrix}$$

Taking (a – b) common from C1 & (b – c) common from C_2

$$= abc(a-b)(b-c)\begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & c \\ a + b & b + c & c^2. \end{vmatrix}$$

$C_1 - C_2$

$$= abc(a-b)(b-c)\begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & c \\ a - c & b + c & c^2 \end{vmatrix}$$

Taking (a – c) common from C_1

$$= abc(a-b)(b-c)(a-c)\begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & c \\ 1 & b + c & c^2 \end{vmatrix}$$

Expanding the determinant

$$= abc(a-b)(b-c)(a-c)\Big(1(0 - 1)\Big)$$

$$= abc(a-b)(b-c)(a-c)\ \ (-1)$$

$$= abc(a-b)(b-c)(c-a)$$

05. $\begin{vmatrix} x & p & q \\ p & x & q \\ p & q & x \end{vmatrix} = (x-p)(x-q)(x+p+q)$

$R_1 - R_2$, $R_2 - R_3$

$$= \begin{vmatrix} x-p & p-x & 0 \\ 0 & x-q & q-x \\ p & q & x \end{vmatrix}$$

Taking $(x - p)$ common from R_1 & $(x - q)$ common from R_2

$$= (x-p)(x-q)\begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ p & q & x \end{vmatrix}$$

Expanding the determinant

$$= (x-p)(x-q)\left(1(x+q) + 1(0+p)\right)$$

$$= (x-p)(x-q)(x+p+q)$$

06. $\begin{vmatrix} a & b+c & a^2 \\ b & c+a & b^2 \\ c & a+b & c^2 \end{vmatrix} = -(a-b)(b-c)(c-a)(a+b+c)$

$C_1 - C_2$

$$= \begin{vmatrix} a+b+c & b+c & a^2 \\ a+b+c & c+a & b^2 \\ a+b+c & a+b & c^2 \end{vmatrix}$$

Taking $(a - c)$ common from C_1

$$= (a+b+c)\begin{vmatrix} 1 & b+c & a^2 \\ 1 & c+a & b^2 \\ 1 & a+b & c^2 \end{vmatrix}$$

$R_1 - R_2$, $R_2 - R_3$

$$= (a+b+c)\begin{vmatrix} 0 & b-a & a^2-b^2 \\ 0 & c-b & b^2-c^2 \\ 1 & a+b & c^2 \end{vmatrix}$$

Taking $(a - b)$ common from R_1 & $(b - c)$ common from R_2

$$= (a+b+c)a-b)(b-c)\begin{vmatrix} 0 & -1 & a+b \\ 0 & -1 & b+c \\ 1 & a+b & c^2 \end{vmatrix}$$

$R_1 - R_2$

$$= (a+b+c)a-b)(b-c)\begin{vmatrix} 0 & 0 & a-c \\ 0 & -1 & b+c \\ 1 & a+b & c^2 \end{vmatrix}$$

Taking $(a - c)$ common from R_1

$$= (a+b+c)a-b)(b-c)(a-c)\begin{vmatrix} 0 & 0 & 1 \\ 0 & -1 & b+c \\ 1 & a+b & c^2 \end{vmatrix}$$

Expanding the determinant

$$= (a+b+c)(a-b)(b-c)(a-c)\left(1(0+1)\right)$$

$$= (a+b+c)(a-b)(b-c)(a-c)\ (1)$$

$$= -(a+b+c)(a-b)(b-c)(c-a)$$

07. $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$

$R_1 - R_2$, $R_2 - R_3$

$$= \begin{vmatrix} a+b+c & -a-b-c & 0 \\ 0 & b+c+a & -b-c-a \\ c & a & c+a+2b \end{vmatrix}$$

Taking $(a+b+c)$ common from R_1 & R_2 respectively .

$$= (a+b+c)^2\begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ c & a & c+a+2b \end{vmatrix}$$

Expanding the determinant

$$= (a+b+c)^2\ \left(1(c+a+2b+a) + 1(0+c)\right)$$

$$= (a+b+c)^2\quad(2a+2b+2c)$$

$$= (a+b+c)^2\quad 2(a+b+c)$$

$$= 2(a+b+c)^3$$

08. $$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$

$C_1 - C_2$

$$= \begin{vmatrix} -a-b-c & 0 & 2a \\ b+c+a & -b-c-a & 2b \\ 0 & c+a+b & c-a-b \end{vmatrix}$$

Taking (a+b+c) common from R_1 & R_2 respectively .

$$= (a+b+c)^2 \begin{vmatrix} -1 & 0 & 2a \\ 1 & -1 & 2b \\ 0 & 1 & c-a-b \end{vmatrix}$$

Expanding the determinant

$$= (a+b+c)^2 \left[-1(-c+a+b-2b) + 2a(1+0) \right]$$

$$= (a+b+c)^2 (c-a-b+2b+2a)$$

$$= (a+b+c)^2 (a+b+c)$$

$$= (a+b+c)^3$$

09. $$\begin{vmatrix} 1 & x & x^2 - yz \\ 1 & y & y^2 - zx \\ 1 & z & z^2 - xy \end{vmatrix} = 0$$

$R_1 - R_2$, $R_2 - R_3$

$$= \begin{vmatrix} 0 & x-y & x^2 - y^2 - yz + zx \\ 0 & y-z & y^2 - z^2 - zx + xy \\ 1 & z & z^2 - xy \end{vmatrix}$$

$$= \begin{vmatrix} 0 & x-y & (x-y)(x+y) + z(x-y) \\ 0 & y-z & (y-z)(y+z) + x(y-z) \\ 1 & z & z^2 - xy \end{vmatrix}$$

Taking (x – y) common from R_1 & (y – z) common from R_2

$$= (x-y)(y-z) \begin{vmatrix} 0 & 1 & x+y+z \\ 0 & 1 & x+y+z \\ 1 & z & z^2 - xy \end{vmatrix}$$

$$= (x-y)(y-z) \quad (0)$$

... Since R1 & R2 are identical

$$= 0 = \text{RHS}$$

10. $$\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$$

$$= (a-b)(b-c)(c-a)(ab+bc+ca)$$

$R_1 - R_2$, $R_2 - R_3$

$$= \begin{vmatrix} 0 & a^2 - b^2 & a^3 - b^3 \\ 0 & b^2 - c^2 & b^3 - c^3 \\ 1 & c^2 & c^3 \end{vmatrix}$$

Taking (a – b) common from R_1 & (b – c) common from R_2

$$= (a-b)(b-c) \begin{vmatrix} 0 & a+b & a^2+ab+b^2 \\ 0 & b+c & b^2+bc+c^2 \\ 1 & c^2 & c^3 \end{vmatrix}$$

$R_1 - R_2$

$$= (a-b)(b-c) \begin{vmatrix} 0 & a-c & a^2-c^2+ab-bc \\ 0 & b+c & b^2+bc+c^2 \\ 1 & c^2 & c^3 \end{vmatrix}$$

$$= (a-b)(b-c) \begin{vmatrix} 0 & a-c & (a-c)(a+c)+b(a-c) \\ 0 & b+c & b^2+bc+c^2 \\ 1 & c^2 & c^3 \end{vmatrix}$$

Taking (a – c) common from R_1

$$= (a-b)(b-c)(a-c) \begin{vmatrix} 0 & 1 & a+b+c \\ 0 & b+c & b^2+bc+c^2 \\ 1 & c^2 & c^3 \end{vmatrix}$$

Expanding the determinant

$$= (a-b)(b-c)(a-c).$$
$$\left[-1(0-b^2-bc-c^2) + (a+b+c)(0-b-c) \right]$$

$= (a-b)(b-c)(a-c).$
$(b^2+bc+c^2 -ab-ac-c^2-b^2-bc-bc-c^2)$

$= (a-b)(b-c)(a-c).(-ab-bc-ac)$

$= (a-b)(b-c)(c-a).(ab+bc+ac)$

$=$ RHS

11. $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^3 & y^3 & z^3 \end{vmatrix} = (x-y)(y-z)(z-x)(x+y+z)$

$C_1 - C_2$, $C_2 - C_3$

$$= \begin{vmatrix} 0 & 0 & 1 \\ x-y & y-z & z \\ x^3-y^3 & y^3-z^3 & z^3 \end{vmatrix}$$

Taking (x – y) common from C_1 & (y – z) common from C_2

$$= (x-y)(y-z)\begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & z \\ x^2+xy+y^2 & y^2+yz+z^2 & z^3 \end{vmatrix}$$

$C_1 - C_2$

$$= (x-y)(y-z)\begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & z \\ x^2-z^2+xy-yz & y^2+yz+z^2 & z^3 \end{vmatrix}$$

$$= (x-y)(y-z)\begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & z \\ (x-z)(x+z)+y(x-z) & y^2+yz+z^2 & z^3 \end{vmatrix}$$

$$= (x-y)(y-z)\begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & z \\ (x-z)(x+y+z) & y^2+yz+z^2 & z^3 \end{vmatrix}$$

Expanding the determinant

$= (x-y)(y-z) \quad 1(\,0 - (x-z)(x+y+z)\,)$

$= (x-y)(y-z)(z-x)(x+y+z)$

12. $\begin{vmatrix} y+z & z & y \\ z & z+x & x \\ y & x & x+y \end{vmatrix} = 4xyz$

$C_1 - C_2$

$$= \begin{vmatrix} y & z & y \\ -x & z+x & x \\ y-x & x & x+y \end{vmatrix}$$

$C_1 + C_3$

$$= \begin{vmatrix} 2y & z & y \\ 0 & z+x & x \\ 2y & x & x+y \end{vmatrix}$$

Taking '2' common from C_1

$$= 2\begin{vmatrix} y & z & y \\ 0 & z+x & x \\ y & x & x+y \end{vmatrix}$$

$C_3 - C_1$

$$= 2\begin{vmatrix} y & z & 0 \\ 0 & z+x & x \\ y & x & x \end{vmatrix}$$

$C_2 - C_3$

$$= 2\begin{vmatrix} y & z & 0 \\ 0 & z & x \\ y & 0 & x \end{vmatrix}$$

Taking 'x' , 'y' & z common from C_1,C_2 & C_3 respectively

$$= 2xyz\begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix}$$

$= 2xyz\,\left[1(1-0) - 1(0-1)\right]$

$= 2xyz(1+1) = 4xyz$

13. **WITHOUT EXPANSION , PROVE**

$$\begin{vmatrix} 6 & 1 & 2 \\ 7 & 2 & 3 \\ 2 & 3 & -4 \end{vmatrix} + \begin{vmatrix} 2 & 4 & 7 \\ 3 & 4 & 2 \\ -4 & 3 & 3 \end{vmatrix} = 5\begin{vmatrix} 2 & 2 & 7 \\ 3 & 1 & 2 \\ -4 & 1 & 3 \end{vmatrix}$$

LHS

$$= \begin{vmatrix} 6 & 1 & 2 \\ 7 & 2 & 3 \\ 2 & 3 & -4 \end{vmatrix} + \begin{vmatrix} 2 & 4 & 7 \\ 3 & 4 & 2 \\ -4 & 3 & 3 \end{vmatrix}$$

R ↔ C **(ONLY ON THE FIRST DETERMINANT)**

$$= \begin{vmatrix} 6 & 7 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & -4 \end{vmatrix} + \begin{vmatrix} 2 & 4 & 7 \\ 3 & 4 & 2 \\ -4 & 3 & 3 \end{vmatrix}$$

$C_2 \leftrightarrow C_3$ **(ONLY ON THE FIRST DETERMINANT)**

$$= -\begin{vmatrix} 6 & 2 & 7 \\ 1 & 3 & 2 \\ 2 & -4 & 3 \end{vmatrix} + \begin{vmatrix} 2 & 4 & 7 \\ 3 & 4 & 2 \\ -4 & 3 & 3 \end{vmatrix}$$

$C_1 \leftrightarrow C_2$ **(ONLY ON THE FIRST DETERMINANT)**

$$= \begin{vmatrix} 2 & 6 & 7 \\ 3 & 1 & 2 \\ -4 & 2 & 3 \end{vmatrix} + \begin{vmatrix} 2 & 4 & 7 \\ 3 & 4 & 2 \\ -4 & 3 & 3 \end{vmatrix}$$

$$= \begin{vmatrix} 2 & 6+4 & 7 \\ 3 & 1+4 & 2 \\ -4 & 2+3 & 3 \end{vmatrix}$$

$$= \begin{vmatrix} 2 & 10 & 7 \\ 3 & 5 & 2 \\ -4 & 5 & 3 \end{vmatrix}$$

taking '5' common from C_2

$$= 5\begin{vmatrix} 2 & 2 & 7 \\ 3 & 1 & 2 \\ -4 & 1 & 3 \end{vmatrix} = \text{RHS}$$

14. **WITHOUT EXPANSION , PROVE**

$$\begin{vmatrix} 2 & -3 & 4 \\ 5 & -6 & 2 \\ -3 & 1 & 3 \end{vmatrix} + \begin{vmatrix} 2 & 5 & -3 \\ 5 & 8 & 5 \\ 4 & 2 & 3 \end{vmatrix} = -2\begin{vmatrix} 2 & 4 & 1 \\ 5 & 2 & 1 \\ -3 & 3 & 3 \end{vmatrix}$$

LHS

$$= \begin{vmatrix} 2 & -3 & 4 \\ 5 & -6 & 2 \\ -3 & 1 & 3 \end{vmatrix} + \begin{vmatrix} 2 & 5 & -3 \\ 5 & 8 & 5 \\ 4 & 2 & 3 \end{vmatrix}$$

R ↔ C **(ONLY ON THE SECOND DETERMINANT)**

$$= \begin{vmatrix} 2 & -3 & 4 \\ 5 & -6 & 2 \\ -3 & 1 & 3 \end{vmatrix} + \begin{vmatrix} 2 & 5 & 4 \\ 5 & 8 & 2 \\ -3 & 5 & 3 \end{vmatrix}$$

$$= \begin{vmatrix} 2 & -3+5 & 4 \\ 5 & -6+8 & 2 \\ -3 & 1+5 & 3 \end{vmatrix}$$

$$= \begin{vmatrix} 2 & 2 & 4 \\ 5 & 2 & 2 \\ -3 & 6 & 3 \end{vmatrix}$$

taking '2' common from C_2

$$= 2\begin{vmatrix} 2 & 1 & 4 \\ 5 & 1 & 2 \\ -3 & 3 & 3 \end{vmatrix}$$

$C_2 \leftrightarrow C_3$ **(ONLY ON THE SECOND DETERMINANT)**

$$= -2\begin{vmatrix} 2 & 4 & 1 \\ 5 & 2 & 1 \\ -3 & 3 & 3 \end{vmatrix}$$

= RHS

01. $x + y = 3$; $5x + 6y = 17$; $2x - 3y = k$ are consistent . Find k

$x + y = 3$

$5x + 6y = 17$

$2x - 3y = k$ are consistent

Hence

$$\begin{array}{ccc} + & - & + \end{array}$$
$$\begin{vmatrix} 1 & 1 & 3 \\ 5 & 6 & 17 \\ 2 & -3 & k \end{vmatrix} = 0$$

$1(6k + 51) - 1(5k - 34) + 3(-15 - 12) = 0$

$6k + 51 - 5k + 34 + 3(-27) = 0$

$k + 85 - 81 = 0$

$k + 4 = 0$

$k = -4$

02. $3x + y = 2$; $kx + 2y = 3$; $2x - y = -3$ are consistent . Find k

$3x + y = 2$

$kx + 2y = 3$

$2x - y = -3$ are consistent

Hence

$$\begin{array}{ccc} + & - & + \end{array}$$
$$\begin{vmatrix} 3 & 1 & 2 \\ k & 2 & 3 \\ 2 & -1 & -3 \end{vmatrix} = 0$$

$3(-6 + 3) - 1(-3k - 6) + 2(-k - 4) = 0$

$3(-3) + 3k + 6 - 2k - 8 = 0$

$-9 + k - 2 = 0$

$-11 + k = 0$

$k = 11$

03. $x + 3y + 2 = 0$; $4y + 2x = k$; $x - 2y = 3k$. are consistent . Find k

$x + 3y = -2$

$2x + 4y = k$

$x - 2y = 3k$ are consistent

Hence

$$\begin{array}{ccc} + & - & + \end{array}$$
$$\begin{vmatrix} 1 & 3 & -2 \\ 2 & 4 & k \\ 1 & -2 & 3k \end{vmatrix} = 0$$

$1(12k + 2k) - 3(6k - k) - 2(-4 - 4) = 0$

$!(14k) - 18k + 3k - 2(-8) = 0$

$14k - 18k + 3k + 16 = 0$

$-k + 16 = 0$

$k = 16$

04. $2x - y + 3 = 0$; $7x - 2y + 2 = 0$; $kx - y - 1 = 0$ are consistent . Find k & SS

$2x - y = -3$

$7x - 2y = -2$

$kx - y = 1$ are consistent

Hence

$$\begin{array}{ccc} + & - & + \end{array}$$
$$\begin{vmatrix} 2 & -1 & -3 \\ 7 & -2 & -2 \\ k & -1 & 1 \end{vmatrix} = 0$$

$2(-2 - 2) + 1(7 + 2k) - 3(-7 + 2k) = 0$

$2(-4) + 7 + 2k + 21 - 6k = 0$

$-8 + 28 - 4k = 0$

$20 - 4k = 0$

$20 = 4k$

$k = 5$

FOR SOLUTION SET

$$D = \begin{vmatrix} 2 & -1 \\ 7 & -2 \end{vmatrix} = -4 + 7 = 3$$

$$Dx = \begin{vmatrix} -3 & -1 \\ -2 & -2 \end{vmatrix} = 6 - 2 = 4$$

$$Dy = \begin{vmatrix} 2 & -3 \\ 7 & -2 \end{vmatrix} = -4 + 21 = 17$$

$$x = \frac{Dx}{D} = \frac{4}{3} \; ; \; y = \frac{Dy}{D} = \frac{17}{3} \qquad SS : \left\{ \frac{4}{3} , \frac{17}{3} \right\}$$

05. $3x + y = 2$; $kx + 2y = 3$; $2x - y = -3$.

Find k if the lines are concurrent . Hence find the point of concurrency

$3x + y = 2$

$kx + 2y = 3$

$2x - y = -3$ are concurrent

Hence

$$\begin{vmatrix} \overset{+}{3} & \overset{-}{1} & \overset{+}{2} \\ k & 2 & 3 \\ 2 & -1 & -3 \end{vmatrix} = 0$$

$3(-6 + 3) - 1(-3k - 6) + 2(-k - 4) = 0$

$3(-3) + 3k + 6 - 2k - 8 = 0$

$-9 + k - 2 = 0$

$-11 + k = 0$

$k = 11$

FOR SOLUTION SET

$3x + y = 2$

$2x - y = -3$

$$D = \begin{vmatrix} 3 & 1 \\ 2 & -1 \end{vmatrix} = -3 - 2 = -5$$

$$Dx = \begin{vmatrix} 2 & 1 \\ -3 & -1 \end{vmatrix} = -2 + 3 = 1$$

$$Dy = \begin{vmatrix} 3 & 2 \\ 2 & -3 \end{vmatrix} = -9 - 4 = -13$$

$x = \frac{Dx}{D} = \frac{-1}{5}$; $y = \frac{Dy}{D} = \frac{13}{5}$ SS : $\left\{ \frac{-1}{5} , \frac{13}{5} \right\}$

06. $kx + 3y + 4 = 0$; $x + ky + 3 = 0$;

$3x + 4y + 5 = 0$ are consistent .

Find k . Hence find common solution set for smallest value of k

$kx + 3y = -4$

$x + ky = -3$

$3x + 4y = -5$ are consistent

$$\begin{vmatrix} k & 3 & -4 \\ 1 & k & -3 \\ 3 & 4 & -5 \end{vmatrix} = 0$$

taking – common from C_1

$$\begin{vmatrix} \overset{+}{k} & \overset{-}{3} & \overset{+}{4} \\ 1 & k & 3 \\ 3 & 4 & 5 \end{vmatrix} = 0$$

$k(5k - 12) - 3(5 - 9) + 4(4 - 3k)) = 0$

$5k^2 - 12k - 3(-4) + 16 - 12k = 0$

$5k^2 - 12k + 12 + 16 - 12k = 0$

$5k^2 - 24k + 28 = 0$

$5k^2 - 10k - 14k + 28 = 0$

$5k(k - 2) - 14(k - 2) = 0$

$k = 14/5$, $k = 2$

SOLUTION SET FOR $k = 2$

$2x + 3y = -4$

$x + 2y = -3$

FOR SOLUTION SET

$$D = \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} = 4 - 3 = 1$$

$$Dx = \begin{vmatrix} -4 & 3 \\ -3 & 2 \end{vmatrix} = -8 + 9 = 1$$

$$Dy = \begin{vmatrix} 2 & -4 \\ 1 & -3 \end{vmatrix} = -6 + 4 = -2$$

$x = \frac{Dx}{D} = \frac{1}{1}$; $y = \frac{Dy}{D} = \frac{-2}{1}$ SS :{1 ,–2}

07. $(k-2)x + (k-1)y = 17$

$(k-1)x + (k-2)y = 18$

$x + y = 5$ are consistent . Find k

$$\begin{vmatrix} k-2 & k-1 & 17 \\ k-1 & k-2 & 18 \\ 1 & 1 & 5 \end{vmatrix} = 0$$

$R_1 - R_2$

$$\begin{vmatrix} -1 & 1 & -1 \\ k-1 & k-2 & 18 \\ 1 & 1 & 5 \end{vmatrix} = 0$$

Expanding the determinant

$-1(5k-10-18) - 1(5k-5-18) - 1(k-1-k+2) = 0$

$-1(5k-28) - 1(5k-23) - 1(1) = 0$

$-5k + 28 - 5k + 23 - 1 = 0$

$-10k + 40 = 0$

$10k = 40 \qquad k = 4$

08. $(k+1)x + (k-1)y + (k-1) = 0$;

$(k-1)x + (k+1)y + (k-1) = 0$;

$(k-1)x + (k-1)y + (k+1) = 0$

are consistent . Find k

$$\begin{vmatrix} k+1 & k-1 & -(k-1) \\ k-1 & k+1 & -(k-1) \\ k-1 & k-1 & -(k+1) \end{vmatrix} = 0$$

taking '–' common from C_3

$$\begin{vmatrix} k+1 & k-1 & k-1 \\ k-1 & k+1 & k-1 \\ k-1 & k-1 & k+1 \end{vmatrix} = 0$$

('–' SIGN IS PUSHED INTO RHS)

$R_1 - R_2$, $R_2 - R_3$

$$\begin{vmatrix} 2 & -2 & 0 \\ 0 & 2 & -2 \\ k-1 & k-1 & k+1 \end{vmatrix} = 0$$

taking '2' common from R_1 & R_2

$$\begin{array}{ccc} + & - & + \end{array}$$
$$\begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ k-1 & k-1 & k+1 \end{vmatrix} = 0$$

('4' IS PUSHED INTO RHS)

Expanding the determinant

$1(k + 1 + k - 1) + 1(0 + k - 1) = 0$

$2k + k - 1 = 0$

$3k = 1 \qquad k = 1/3$

09. $x + y + k = 0$; $kx + 2ky + 6 = 0$;

$x + ky + 1 = 0$ are consistent . Find k

$x + y = -k$

$kx + 2ky = -6$

$x + ky = -1$ are consistent

$$\begin{vmatrix} 1 & 1 & -k \\ k & 2k & -6 \\ 1 & k & -1 \end{vmatrix} = 0$$

taking '–' common from C_3

$$\begin{vmatrix} 1 & 1 & k \\ k & 2k & 6 \\ 1 & k & 1 \end{vmatrix} = 0$$

$R_1 - R_3$

$$\begin{vmatrix} 0 & 1-k & k-1 \\ k & 2k & 6 \\ 1 & k & 1 \end{vmatrix} = 0$$

taking 'k – 1' common from R_1

$$(k-1)\begin{vmatrix} 0 & -1 & 1 \\ k & 2k & 6 \\ 1 & k & 1 \end{vmatrix} = 0$$

Expanding the determinant

$$(k-1)\left[+1(k-6)+1(k^2-2k)\right] = 0$$

$$(k-1)(k-6+k^2-2k) = 0$$

$$(k-1)(k^2-k-6) = 0$$

$$(k-1)(k^2-3k+2k-6) = 0$$

$$(k-1)(k-3)(k+2) = 0$$

$k = 1$; $k = 3$; $k = -2$

10. $(k+2)x + 2y = 6$; $5x + (k-1)y = 6$; $5x + 2y = k + 3$. are consistent . Find k

$$\begin{aligned}(k+2)x + 2y &= 6\\ 5x + (k-1)y &= 6\\ 5x + 2y &= k+3\end{aligned}$$

are consistent , hence

$$\begin{vmatrix} k+2 & 2 & 6 \\ 5 & k-1 & 6 \\ 5 & 2 & k+3 \end{vmatrix} = 0$$

$R_1 - R_2$; $R_2 - R_3$

$$\begin{vmatrix} k-3 & 3-k & 0 \\ 0 & k-3 & 3-k \\ 5 & 2 & k+3 \end{vmatrix} = 0$$

taking 'k – 3' common from R1 & R2 respt.

$$(k-3)^2 \begin{array}{c} \begin{array}{ccc} + & - & + \end{array} \\ \begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 5 & 2 & k+3 \end{vmatrix} \end{array} = 0$$

Expanding the determinant

$$(k-3)^2\left[1(k+3+2)+1(0+5)\right] = 0$$

$$(k-3)^2\ (k+5+5) = 0$$

$$(k-3)^2(k+10) = 0$$

$k = 3$; $k = -10$

11. $2kx - 3y = 8$; $4x - (k+1)y = 8$; $4x - 3y = 3k + 2$. are consistent . Find k

$$\begin{aligned}2kx - 3y &= 8\\ 4x - (k+1)y &= 8\\ 4x - 3y &= 3k+2\end{aligned}$$ are consistent

$$\begin{vmatrix} 2k & -3 & 8 \\ 4 & -(k+1) & 8 \\ 4 & -3 & 3k+2 \end{vmatrix} = 0$$

Taking '2' common from C_1 & '–' common from C_2

$$\begin{vmatrix} k & 3 & 8 \\ 2 & k+1 & 8 \\ 2 & 3 & 3k+2 \end{vmatrix} = 0$$

$R_1 - R_2$, $R_2 - R_3$

$$\begin{vmatrix} k-2 & 2-k & 0 \\ 0 & k-2 & 6-3k \\ 2 & 3 & 3k+2 \end{vmatrix} = 0$$

$$\begin{vmatrix} k-2 & 2-k & 0 \\ 0 & k-2 & 3(2-k) \\ 2 & 3 & 3k+2 \end{vmatrix} = 0$$

taking 'k – 2' common from R1 & R2 respt.

$$(k-2)^2 \begin{array}{c} \begin{array}{ccc} + & - & + \end{array} \\ \begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -3 \\ 2 & 3 & 3k+2 \end{vmatrix} \end{array} = 0$$

Expanding the determinant

$$(k-2)^2\ 1(3k+2+9)+1(0+6) = 0$$

$$(k-2)^2\ (3k+11+6) = 0$$

$$(k-2)^2\ (3k+17) = 0$$

$k = 2$; $k = -17/3$

12. if the equations $ax + by + c = 0$; $cx + ay + b = 0$; and $bx + cy + a = 0$ are consistent in x and y then show that $a^3 + b^3 + c^3 = 3abc$

$ax + by = -c$

$cx + ay = -b$

$bx + cy = -a$ are consistent

Hence

$$\begin{vmatrix} a & b & -c \\ c & a & -b \\ b & c & -a \end{vmatrix} = 0$$

taking '–' common from C_1

$$\begin{array}{ccc} + & - & + \end{array}$$
$$\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix} = 0$$

Expanding the determinant

$a(a^2 - bc) - b(ac - b^2) + c(c^2 - ab) = 0$

$a^3 - abc - abc + b^3 + c^3 - abc = 0$

$a^3 + b^3 + c^3 - 3abc = 0$

$a^3 + b^3 + c^3 = 3abc$ ……. PROVED

13. if $l = \frac{y-1}{x}$; $m = \frac{1-x}{y}$; $n = x - y$ are consistent in x and y then prove : $l + m + n + lmn = 0$

$lx - y = -1$

$x + my = 1$

$x - y = n$ are consistent , hence

$$\begin{array}{ccc} + & - & + \end{array}$$
$$\begin{vmatrix} l & -1 & -1 \\ 1 & m & 1 \\ 1 & -1 & n \end{vmatrix} = 0$$

Expanding the determinant

$l(mn + 1) + 1(n - 1) - 1(-1 - m) = 0$

$lmn + l + n - 1 + 1 + m = 0$

$lmn + l + m + n = 0$

14. Show that the equations

$x + ay + (a^2 - bc) = 0$;

$x + by + (b^2 - ca) = 0$

$x + cy + (c^2 - ab) = 0$

are consistent for all a , b , c

$$\begin{vmatrix} 1 & a & a^2 - bc \\ 1 & b & b^2 - ca \\ 1 & c & c^2 - ab \end{vmatrix}$$

$R_1 - R_2$, $R_2 - R_3$

$$= \begin{vmatrix} 0 & a - b & a^2 - b^2 - bc + ca \\ 0 & b - c & b^2 - c^2 - ca + ab \\ 1 & c & c^2 - ab \end{vmatrix}$$

$$= \begin{vmatrix} 0 & a - b & (a - b)(a+b) + c(a - b) \\ 0 & b - c & (b - c)(b+c) + a(b - c) \\ 1 & c & c^2 - ab \end{vmatrix}$$

Taking (a – b) common from R_1 & (b – c) common from R_2

$$= (a-b)(b-c) \begin{vmatrix} 0 & 1 & a + b + c \\ 0 & 1 & a + b + c \\ 1 & c & c^2 - ab \end{vmatrix}$$

$= (a-b)(b-c)\ (0)$

… Since R_1 & R_2 are identical

$= 0$.

Hence given set of equations are consistent

Q1. Find area of triangle with vertices as

01. (4,5) ; (0,7) ; (–1,1)

$$A = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 4 & 5 & 1 \\ 0 & 7 & 1 \\ -1 & 1 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[4(7-1) - 5(0+1) + 1(0+7)\right]$$

$$= \frac{1}{2}\left[24 - 5 + 7\right]$$

$$= \frac{1}{2}(26) \quad = 13 \text{ sq. units}$$

02. (3,6) ; (–1,3) ; (2,–1)

$$A = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 3 & 6 & 1 \\ -1 & 3 & 1 \\ 2 & -1 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[3(3+1) - 6(-1-2) + 1(1-6)\right]$$

$$= \frac{1}{2}\left[12 + 18 - 5\right]$$

$$= \frac{1}{2}(25) \quad = \frac{25}{2} \text{ sq. units}$$

03. (1,1) , (3,7) , (10,8)

$$A = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ 3 & 7 & 1 \\ 10 & 8 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[1(7-8) - 1(3-10) + 1(24-70)\right]$$

$$= \frac{1}{2}\left[-1 + 7 - 46\right]$$

$$= \frac{1}{2}(-40)$$

$$= -20 \qquad = 20 \text{ sq. units}$$

Q2.

01. Find k if the area of the triangle whose vertices are A (4 , k) ; B (–5 , –7); C(–4 , 1) is 38 sq. units

$A\ (\Delta ABC) = 38$

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm 38$$

$$\begin{vmatrix} 4 & k & 1 \\ -5 & -7 & 1 \\ -4 & 1 & 1 \end{vmatrix} = \pm 76$$

$$4(-7-1) - k(-5+4) + 1(-5-28) = \pm 76$$

$$4(-8) - k(-1) + 1(-33) = \pm 76$$

$$-32 + k - 33 = \pm 76$$

$$-65 + k = \pm 76$$

$-65 + k = 76$ OR $-65 + k = -76$

$k = 141$ $\qquad$ $k = -11$

02. Find k if the area of the triangle whose vertices are P (k , –4) ; Q (1 ,–2) ; R (4 ,– 5) is 15⁄2 sq. units

$A\ (\Delta ABC) = 15/2$

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm \frac{15}{2}$$

$$\begin{vmatrix} k & -4 & 1 \\ 1 & -2 & 1 \\ 4 & -5 & 1 \end{vmatrix} = \pm 15$$

$$k(-2+5) + 4(1-4) + 1(-5+8) = \pm 15$$

$$3k - 12 + 3 = \pm 15$$

$3k - 9 = \pm 15$

$3k - 9 = 15$ OR $3k - 9 = -15$

$3k = 24$ $\quad$ $3k = -6$

$k = 8$ OR $k = -2$

03. Find k if the area of the triangle whose vertices are P (3 , –5) ; Q (–2 , k) ; R (1 , 4) is 33⁄2 sq. units

A (ΔABC) = 33/2

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm \frac{33}{2}$$

$$\begin{vmatrix} 3 & -5 & 1 \\ -2 & k & 1 \\ 1 & 4 & 1 \end{vmatrix} = \pm 33$$

$3(k - 4) + 5(-2 - 1) + 1(-8 - k) = \pm 33$

$3k - 12 - 15 - 8 - k = \pm 33$

$2k - 35 = \pm 33$

$2k - 35 = 33$ OR $2k - 35 = -33$

$2k = 68$ $\quad$ $2k = 2$

$k = 34$ OR $k = 1$

Q3. Find area of Quadrilateral whose vertices are A (2 , 1) ; B (2 , 3) ; C(–2 , 2) ; D(–1 , 0)

$$A(\Delta ABC) = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 2 & 1 & 1 \\ 2 & 3 & 1 \\ -2 & 2 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[2(3-2) - 1(2+2) + 1(4+6)\right]$$

$$= \frac{1}{2}\left[2 - 4 + 10\right]$$

$$= \frac{1}{2}(8) = 4 \text{ sq. units}$$

$$A(\Delta ADC) = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 2 & 1 & 1 \\ -1 & 0 & 1 \\ -2 & 2 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[2(0-2) - 1(-1+2) + 1(-2+0)\right]$$

$$= \frac{1}{2}\left[-4 - 1 - 2\right]$$

$$= -\frac{7}{2} = \frac{7}{2} \text{ sq. units}$$

$$A(\square ABCD) = A(\Delta ABC) + A(\Delta ADC)$$

$$= 4 + \frac{7}{2}$$

$$= {}^{15}/_{2} \text{ sq. units}$$

Q4.

Using determinants show that the following set of points are collinear

01. A(3,1) ; B(4,2) ; C(5, 3)

$$A = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 3 & 1 & 1 \\ 4 & 2 & 1 \\ 5 & 3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[3(2-3) - 1(4-5) + 1(12-10)\right]$$

$$= \frac{1}{2}\left[-3 + 1 + 2\right]$$

$$= 0$$

Hence the points are collinear

02. A(1,–2) ; B(3,1) ; C(5,4)

$$A = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 1 & -2 & 1 \\ 3 & 1 & 1 \\ 5 & 4 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[1(1-4) + 2(3-5) + 1(12-5)\right]$$

$$= \frac{1}{2}\left[-3 - 4 + 7\right]$$

$$= 0 \ .$$

Hence the points are collinear

03. A(3,7) ; B(4,–3) ; C(5,–13)

$$A = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 3 & 7 & 1 \\ 4 & -3 & 1 \\ 5 & -13 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[3(-3+13) - 7(4-5) + 1(-52+15)\right]$$

$$= \frac{1}{2}\left[3(10) - 7(-1) - 1(37)\right]$$

$$= \frac{1}{2}\left[30 + 7 - 37\right]$$

$$= 0$$

Hence the points are collinear

04.

points A(a,0) ; B(0,b) , C(1,1) are collinear .

Prove : $\frac{1}{a} + \frac{1}{b} = 1$

Since points are collinear ;

$A(\Delta ABC) = 0$

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

$$\frac{1}{2}\begin{vmatrix} a & 0 & 1 \\ 0 & b & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

$$a(b-1) - 0(0-1) + 1(0-b) = 0$$

$$ab - a - b = 0$$

$$ab = a + b$$

Dividing throughout by ab

$$1 = \frac{a}{ab} + \frac{b}{ab}$$

$$\frac{1}{a} + \frac{1}{b} = 1 \qquad \ldots\ldots\ldots \quad \text{PROVED}$$

05 if points A(a ,b) ;B(c , d) ; C(a – c , b – d) are collinear then prove that ad – bc = 0

Since points are collinear ;

$A(\Delta ABC) = 0$

$$\frac{1}{2}\begin{vmatrix} a & b & 1 \\ c & d & 1 \\ a-c & b-d & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} a & b & 1 \\ c & d & 1 \\ a-c & b-d & 1 \end{vmatrix} = 0$$

$$a(d-b+d) - b(c-a+c) + 1(bc-cd-ad+cd) = 0$$

$$a(2d-b) - b(2c-a) + 1(bc-ad) = 0$$

$$2ad - ab - 2bc + ab + bc - ad = 0$$

$$ad - bc = 0$$